

Model-independent determination of the parity of Ξ hyperons

K. Nakayama,^{1,2,*} Yongseok Oh,^{3,4,†} and H. Haberzettl^{5,‡}

¹*Department of Physics and Astronomy, University of Georgia, Athens, Georgia 30602, USA*

²*Institut für Kernphysik and Center for Hadron Physics, Forschungszentrum Jülich, D-52425 Jülich, Germany*

³*Department of Physics, Kyungpook National University, Daegu 702-701, Korea*

⁴*Asia Pacific Center for Theoretical Physics, Pohang, Gyeongbuk 790-784, Korea*

⁵*Institute for Nuclear Studies and Department of Physics, The George Washington University, Washington, DC 20052, USA*

(Received 20 February 2012; published 6 April 2012)

Based on reflection symmetry in the reaction plane, it is shown that measuring the transverse spin-transfer coefficient K_{yy} in the $\bar{K}N \rightarrow K\Xi$ reaction directly determines the parity of the produced cascade hyperon in a model-independent way as $\pi_{\Xi} = K_{yy}$, where $\pi_{\Xi} = \pm 1$ is the parity. This result based on Bohr's theorem provides a completely general, universal relationship that applies to the entire hyperon spectrum. A similar expression is obtained for the photoreaction $\gamma N \rightarrow K K\Xi$ by measuring both the double-polarization observable K_{yy} and the photon-beam asymmetry Σ . Regarding the feasibility of such experiments, it is pointed out that the self-analyzing property of the Ξ s can be invoked, thus requiring only a polarized nucleon target.

DOI: [10.1103/PhysRevC.85.042201](https://doi.org/10.1103/PhysRevC.85.042201)

PACS number(s): 13.75.Jz, 13.60.Rj, 13.88.+e, 14.20.Jn

The spectrum of multistrangeness hyperons is largely unknown and much is yet to be explored. For example, the flavor SU(3) symmetry leads to the expectation that the number of Ξ resonances is equal to the number of nonstrange baryon resonances (i.e., the nucleon and Δ resonances). However, the compilation of particle data found in the Particle Data Group Review (PDG) [1] shows that, to date, only a dozen Ξ s have been discovered compared to about 40 nonstrange baryon resonances. Furthermore, only two of them, $\Xi(1318)$ and $\Xi(1530)$, have four-star status according to the PDG. One of the reasons for this situation is that the Ξ hyperons, being particles with strangeness $S = -2$, can only be produced via indirect processes from the nucleon, with very small production yields that makes them difficult to measure. Moreover, the situation is exacerbated by the lack of facilities that can produce antikaon beams. As a result, nothing of significance regarding Ξ resonances has been added to the PDG listings during the last two decades [1]. The situation is going to change very soon with the availability of the antikaon beam at the Japan Proton Accelerator Research Complex (J-PARC) facility which has started its operation just recently. Since the antikaon has strangeness $S = -1$, hyperons with $S = -2$ (Ξ) can be produced directly in reactions such as $\bar{K}N \rightarrow K\Xi$ with sufficiently large yields. Indeed, the study of multistrangeness hyperons is one of the major parts of the physics programs at J-PARC [2,3]. Furthermore, the PANDA Collaboration has also proposed an investigation of the $\bar{p}p \rightarrow \Xi\Xi$ reaction at the Facility for Antiproton and Ion Research (FAIR) [4,5]. Also, the CLAS Collaboration at the Thomas Jefferson National Accelerator Facility (JLab) has established the feasibility of investigating Ξ spectroscopy via photoproduction reactions like $\gamma p \rightarrow K^+K^+\Xi^-$ and $\gamma p \rightarrow K^+K^+\pi^-\Xi^0$ [6–8]. A dedicated experiment for these

reactions has been carried out and the data for the total and differential cross sections as well as the K^+K^+ and $K^+\Xi^-$ invariant mass distributions for the $\gamma p \rightarrow K^+K^+\Xi^-$ reaction have been reported in Ref. [9]. Thus, a data set measured for the exclusive production of the Ξ in photon-nucleon scattering now exists.

Theoretical studies of the production of Ξ hyperons also started only recently. For example, the production mechanisms for Ξ photoproduction (i.e., $\gamma p \rightarrow K^+K^+\Xi^-$) was investigated in Refs. [10,11] and recent works for the $\bar{K}N \rightarrow K\Xi$ reaction were reported in Refs. [12,13].

The investigation of multistrangeness baryons is expected to shed light on our understanding of the structure of baryons and it will allow us to distinguish various phenomenological models of the baryon-mass spectrum [14–18]. Knowing the parity quantum number, in particular, is of crucial importance in baryon spectroscopy since it heavily depends on the internal structure of the baryon. However, the experimental extraction of the spin-parity quantum numbers is very difficult. The parity quantum number of the Ξ ground state, $\Xi(1318)$ in particular, has not been measured yet but is assigned to be positive in the PDG compilation based on the quark-model predictions [1]. Therefore, given this uncertain situation, reliable experimental determinations of the quantum numbers of the Ξ ground state and its resonances are important and timely and of particular interest for the experimental programs at facilities that can produce cascades, like J-PARC and others.

There are some earlier efforts to determine the spin-parity quantum numbers of a cascade resonance, in particular of $\Xi^*(1820)$, through an analysis of the moments of its decay products [19–21]. The procedure of Ref. [21] permits the determination of both spin and parity. However, it is limited to resonances above threshold with odd relative orbital angular momentum between the decay products.

In this article, we show an alternative, completely model-independent, and universal way of determining the parity of any Ξ hyperon with an arbitrary spin. This is based on Bohr's theorem [22] which is a consequence of the invariance of the

*nakayama@uga.edu

†yohphy@knu.ac.kr

‡helmut@gwu.edu

transition amplitude under rotation and parity inversion and, in particular, reflection symmetry in the reaction plane. To this end, we consider the reaction $\bar{K}N \rightarrow K\Xi$, where the Ξ hyperon has spin j . This is one of the reactions that will be studied at J-PARC. For completeness, we also consider the parity determination of Ξ via the photoproduction reaction $\gamma N \rightarrow K K \Xi$, which can be performed at JLab.

While completely general, the foremost practical use of the method lies with the ground state of the hyperon, as we shall argue below in the paragraph following Eq. (8). Therefore, we first consider the case of a spin $j = 1/2$ cascade hyperon and show explicitly that the transverse spin-transfer coefficient in Ξ production in the $\bar{K}N$ scattering is directly related to the parity of the Ξ hyperon. We will then generalize the results to the case of a Ξ hyperon with an arbitrary spin j . The most general spin structure of the reaction amplitude, consistent with symmetry principles, for the process $\bar{K}(q) + N(p) \rightarrow K(q') + \Xi(p')$, where the arguments q , p , q' , and p' stand for the four-momenta of the respective particles, is given by

$$\hat{M}^+ = M_0 + M_2 \boldsymbol{\sigma} \cdot \hat{\mathbf{n}}_2, \quad (1a)$$

$$\hat{M}^- = M_1 \boldsymbol{\sigma} \cdot \hat{\mathbf{n}}_1 + M_3 \boldsymbol{\sigma} \cdot \hat{\mathbf{q}}, \quad (1b)$$

for positive and negative parity Ξ (\hat{M}^+ and \hat{M}^-), respectively. Here, $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ is the Cartesian vector made up of the three Pauli matrices σ_i , with indices 1, 2, 3 corresponding to spatial axes x , y , z . The unit vectors $\hat{\mathbf{n}}_1$ and $\hat{\mathbf{n}}_2$ are defined as $\hat{\mathbf{n}}_1 \equiv (\mathbf{q} \times \mathbf{q}') \times \mathbf{q} / |(\mathbf{q} \times \mathbf{q}') \times \mathbf{q}|$ and $\hat{\mathbf{n}}_2 \equiv \mathbf{q} \times \mathbf{q}' / |\mathbf{q} \times \mathbf{q}'|$, respectively.

Without loss of generality, we may choose the coordinate systems such that \mathbf{q} is along the positive z axis and $\hat{\mathbf{n}}_2$ is along the positive y axis. Then $\hat{\mathbf{n}}_1$ is the unit vector along the positive x axis. The plane containing the vectors \mathbf{q} and $\hat{\mathbf{n}}_1$ is the reaction plane and $\hat{\mathbf{n}}_2$ is perpendicular to that plane.

The reaction amplitudes in Eqs. (1) can be summarily written as

$$\hat{M} = \sum_{m=0}^3 M_m \sigma_m, \quad (2)$$

where in addition to the three Pauli matrices σ_i ($i = 1, 2, 3$), σ_0 here is the 2×2 unit matrix. For a positive-parity Ξ , $M_1 = M_3 = 0$ and \hat{M} reduces to \hat{M}^+ , and for a negative-parity Ξ , \hat{M} reduces to \hat{M}^- because $M_0 = M_2 = 0$. Expressing amplitudes utilizing \hat{M} of Eq. (2), the unpolarized cross section is given by

$$\frac{d\sigma}{d\Omega} \equiv \frac{1}{2} \text{Tr}(\hat{M} \hat{M}^\dagger) = \sum_{m=0}^3 |M_m|^2 \quad (3)$$

and the (diagonal) spin-transfer coefficient K_{ii} ($i = 1, 2, 3$) is obtained as

$$\begin{aligned} \frac{d\sigma}{d\Omega} K_{ii} &\equiv \frac{1}{2} \text{Tr}(\hat{M} \sigma_i \hat{M}^\dagger \sigma_i) \\ &= |M_0|^2 + |M_i|^2 - \sum_{k \neq i} |M_k|^2. \end{aligned} \quad (4)$$

In terms of the cross sections, the spin-transfer coefficient K_{ii} is given by

$$K_{ii} = \frac{[d\sigma_i(++)+d\sigma_i(--)] - [d\sigma_i(+-)+d\sigma_i(-+)]}{[d\sigma_i(++)+d\sigma_i(--)] + [d\sigma_i(+-)+d\sigma_i(-+)]}, \quad (5)$$

where $d\sigma_i$ stands for the differential cross section with the polarization of the target nucleon and of the produced cascade along the i direction. The first and second \pm arguments of $d\sigma_i$ indicate the parallel (+) or antiparallel (−) spin-alignment along the i direction of the target nucleon and produced cascade, respectively.

In general, K_{ii} depends on the energy and scattering angle. However, from Eqs. (3) and (4), it follows immediately that K_{yy} is constant and that it provides the parity π_Ξ of Ξ , viz.

$$\pi_\Xi = \pm 1 = K_{yy}, \quad (6)$$

where the sign directly corresponds to positive or negative parity. This result is a direct consequence of the spin structures of the reaction amplitudes for positive and negative parity Ξ as exhibited in Eqs. (1), which, in turn, is a consequence of reflection symmetry in the reaction plane.

The above results can be straightforwardly generalized to a Ξ with an arbitrary spin j by invoking Bohr's theorem [22] written in the form [23]

$$\pi_{fi} = (-1)^{M_f - M_i}. \quad (7)$$

Here, π_{fi} denotes the product of the intrinsic parities of all the particles in the initial (i) and final (f) states, while M_i (M_f) stands for the sum of the spin projection quantum numbers of the initial-state (final-state) particles along the axis perpendicular to the reaction plane, (i.e., $\hat{\mathbf{n}}_2$ or the \hat{y} axis). For the reaction in question, $\pi_{fi} = \pi_\Xi$, and thus

$$\pi_\Xi = (-1)^{M_f - M_i} = K_{yy}. \quad (8)$$

The results given in Eqs. (6) and (8), therefore, directly determine the parity of the produced Ξ hyperon.

To discuss the feasibility of this determination, the present results show that, to obtain the parity of Ξ , one needs to measure the double-polarization observable K_{yy} , which is usually extremely challenging experimentally. The task simplifies considerably if the weak decay modes of the hyperon can be separated from the strong ones, which is particularly true for the ground state where strong decays are absent altogether. One may then employ the fact that the cascade states are self-analyzing under weak decays [24], thus requiring only a polarized nucleon target to determine the polarization of the Ξ s. Such a polarized target may be available at J-PARC in the foreseeable future [25], which would make this experiment possible for the spin-1/2 ground state of Ξ . The feasibility of such an experiment hinges on the cross-section yield with a polarized nucleon target, which should be smaller than the unpolarized cross section by roughly a factor of 10 if one assumes a typical degree of polarization of $\sim 20\%$ of the target nucleon. Since the unpolarized cross section for $K^- p \rightarrow K^+ \Xi^-$ is of the order of $10 \mu\text{b/sr}$ around $\sqrt{s} \sim 2 \text{ GeV}$ [26–31], one might expect cross-section yields of the order of $1 \mu\text{b/sr}$ with the polarized nucleon target.

It should be mentioned that, in principle, for $j = 1/2$ the parity of the cascade resonance may also be determined by measuring single-polarization observables; namely, the target-nucleon asymmetry T_i and the recoil-cascade polarization P_i ,

$$\frac{d\sigma}{d\Omega} T_i \equiv \frac{1}{2} \text{Tr}(M \sigma_i M^\dagger) = 2\text{Re}[M_0 M_i^*] + 2\text{Im}[M_j M_k^*], \quad (9a)$$

$$\frac{d\sigma}{d\Omega} P_i \equiv \frac{1}{2} \text{Tr}(M M^\dagger \sigma_i) = 2\text{Re}[M_0 M_i^*] - 2\text{Im}[M_j M_k^*], \quad (9b)$$

where the subscripts (i, j, k) run cyclically [i.e., (1,2,3), (3,1,2), (2,3,1)]. Then, with the amplitudes given by Eqs. (1), it follows immediately that

$$\frac{d\sigma}{d\Omega} (T_y + P_y) = 4\text{Re}[M_0 M_2^*], \quad (10a)$$

$$\frac{d\sigma}{d\Omega} (T_y - P_y) = 0 \quad (10b)$$

for positive-parity cascade and

$$\frac{d\sigma}{d\Omega} (T_y + P_y) = 0, \quad (11a)$$

$$\frac{d\sigma}{d\Omega} (T_y - P_y) = 4\text{Im}[M_3 M_1^*] \quad (11b)$$

for negative-parity cascade. The equations here reveal that, if the measured combination $T_y + P_y$ is different from zero the parity of the cascade is positive; conversely, if the combination $T_y - P_y$ is different from zero, the parity is negative. Here, it should be noted that the usefulness of these expressions hinges on how reliably the respective right-hand sides of Eqs. (10a) and (11b) can be determined to be different from zero experimentally, which may not be possible if any one of the amplitudes M_i ($i = 0, 1, 2, 3$) is too small. This magnitude problem aside, this experiment would be easier to set up because it requires only measuring the single-polarization observables, T_y and P_y . The determination of the double-polarization observable K_{yy} , by contrast, while requiring a more complex experimental setup, is free of any potential magnitude problem.

The parity of the cascade hyperon may also be determined in a model-independent way in the photoproduction reaction $\gamma N \rightarrow K K \Xi$ that will be studied at JLab [32]. Since kaons are spin-zero particles, in this case one can simply make use of the results derived in Ref. [33].¹ One finds, in particular, among the various spin observables and combinations of spin observables for this reaction that can be used in principle to determine the parity of Ξ , the transverse spin-transfer coefficient with the unpolarized photon beam K_{yy} and the

photon-beam asymmetry Σ are related to the parity of the Ξ resonance by

$$\pi_\Xi = \frac{K_{yy}}{\Sigma}. \quad (12)$$

Obviously, here the measurements of spin observables, especially the double-polarization observable K_{yy} , are more challenging than in hadronic reactions due to much smaller cross-section yields.

The relations found here for the Ξ hyperon can also be applied to the parity determination of the Ω hyperon in the reactions $\bar{K} N \rightarrow K K \Omega$ and $\gamma N \rightarrow K K K \Omega$. Because the production yields are much smaller for the Ω hyperons than those for the Ξ hyperons, the required measurements of polarization observables in Ω production would be much more difficult. At any rate, our results given in Eqs. (6), (8), and (12) can be used to determine the parity of Ω hyperons in principle. However, because of the presence of an additional kaon in Ω production, π_Ξ in these relations should be replaced by $-\pi_\Omega$.

In summary, we have shown that, based on reflection symmetry in the reaction plane, the parity of a Ξ hyperon with an arbitrary spin can be directly determined in a model-independent, universal manner by measuring the transverse spin-transfer coefficient K_{yy} in the $\bar{K} N \rightarrow K \Xi$ reaction that will be studied at the J-PARC facility. Our result is particularly relevant for the ground state of the Ξ since, in this case, one may exploit the fact that the Ξ is self-analyzing under weak decays. In principle, however, our theoretical result applies to the entire cascade spectrum. The parity of the cascade hyperon may also be determined in the photoproduction reaction $\gamma N \rightarrow K K \Xi$, provided one can measure the transverse spin-transfer coefficient with the unpolarized photon beam and the beam asymmetry with linearly polarized photons. We also mention that, since the respective quantities K_{yy} and K_{yy}/Σ for both types of experiments need to be equal to known constants (i.e., $\pi_\Xi = \pm 1$), apart from providing the parity of Ξ , measurements of these quantities also provide some lower limits for the systematic errors of such experiments. Regarding the practical feasibility of such experiments, we mention that the self-analyzing feature of the Ξ s will help if their weak decay modes can be measured to determine the polarization of these Ξ hyperons. Finally, we add that the present discussions can also be applied to the parity determination of Ω hyperons.

We are grateful to Lei Guo, Johann Haidenbauer, Wooyoung Kim, Wim Kloet, Shin'ya Sawada, and Igor Strakovsky for fruitful discussions. This work was partly supported by the National Research Foundation of Korea funded by the Korean Government (Grant No. NRF-2011-220-C00011). The work of K. N. was also supported partly by the FFE Grant No. 41788390 (COSY-058).

¹Details of the derivation of the spin-structure of the photoproduction amplitudes used in Ref. [33], especially those involving a negative-parity baryon, can be found in Ref. [34].

[1] Particle Data Group, K. Nakamura *et al.*, *J. Phys. G* **37**, 075021 (2010) [<http://pdg.lbl.gov>].

[2] T. Nagae, *Lect. Notes Phys.* **724**, 81 (2007).

[3] T. Nagae, *Nucl. Phys. A* **805**, 486c (2008).

[4] T. R. Saitoh and J. Pochodzalla, *Acta Phys. Pol. B* **35**, 1033 (2004).

- [5] W. Erni *et al.* ($\bar{\text{P}}\text{ANDA}$ Collaboration), [arXiv:0903.3905](#).
- [6] J. W. Price *et al.* (CLAS Collaboration), [Phys. Rev. C **71**, 058201 \(2005\)](#).
- [7] J. W. Price, J. Ducote, J. Goetz, and B. M. K. Nefkens (CLAS Collaboration), [Nucl. Phys. A **754**, 272c \(2005\)](#).
- [8] L. Guo and D. P. Weygand (CLAS Collaboration), in *Proceedings of International Workshop on the Physics of Excited Baryons (NSTAR 05)*, edited by S. Capstick, V. Crede, and P. Eugenio (World Scientific, Singapore, 2006), pp. 384–388.
- [9] L. Guo *et al.* (CLAS Collaboration), [Phys. Rev. C **76**, 025208 \(2007\)](#).
- [10] K. Nakayama, Y. Oh, and H. Haberzettl, [Phys. Rev. C **74**, 035205 \(2006\)](#).
- [11] J. K. S. Man, Y. Oh, and K. Nakayama, [Phys. Rev. C **83**, 055201 \(2011\)](#).
- [12] D. A. Sharov, V. L. Korotkikh, and D. E. Lansky, [Eur. Phys. J. A **47**, 109 \(2011\)](#).
- [13] R. Shyam, O. Scholten, and A. W. Thomas, [Phys. Rev. C **84**, 042201 \(2011\)](#).
- [14] Y. Oh, [Phys. Rev. D **75**, 074002 \(2007\)](#).
- [15] K.-T. Chao, N. Isgur, and G. Karl, [Phys. Rev. D **23**, 155 \(1981\)](#).
- [16] S. Capstick and N. Isgur, [Phys. Rev. D **34**, 2809 \(1986\)](#).
- [17] M. Pervin and W. Roberts, [Phys. Rev. C **77**, 025202 \(2008\)](#).
- [18] M. P. Valderrama, J.-J. Xie, and J. Nieves, [Phys. Rev. D **85**, 017502 \(2012\)](#).
- [19] P. Minnaert, [Phys. Lett. B **66**, 157 \(1977\)](#).
- [20] D. Teodoro *et al.*, [Phys. Lett. B **77**, 451 \(1978\)](#).
- [21] S. F. Biagi *et al.*, [Z. Phys. C **34**, 175 \(1987\)](#).
- [22] A. Bohr, [Nucl. Phys. **10**, 486 \(1959\)](#).
- [23] G. R. Satchler, *Direct Nuclear Reactions* (Oxford University Press, New York, 1983).
- [24] G. Källén, *Elementary Particle Physics* (Addison-Wesley, Reading, 1964), Chap. 17; S. Gasiorowicz, *Elementary Particle Physics* (Wiley, New York, 1966), Chap. 32.
- [25] S. Sawada (private communication).
- [26] M. Haque *et al.* (Birmingham-Glasgow-London(I. C.)-Oxford-Rutherford Collaboration), [Phys. Rev. **152**, 1148 \(1966\)](#).
- [27] G. W. London *et al.*, [Phys. Rev. **143**, 1034 \(1966\)](#).
- [28] T. G. Trippe and P. E. Schlein, [Phys. Rev. **158**, 1334 \(1967\)](#).
- [29] W. P. Trower *et al.*, [Phys. Rev. **170**, 1207 \(1968\)](#).
- [30] G. Burgun *et al.*, [Nucl. Phys. B **8**, 447 \(1968\)](#).
- [31] P. M. Dauber *et al.*, [Phys. Rev. **179**, 1262 \(1969\)](#).
- [32] L. Guo (private communication).
- [33] K. Nakayama and W. G. Love, [Phys. Rev. C **70**, 012201 \(2004\)](#).
- [34] K. Nakayama and W. G. Love, [Phys. Rev. C **72**, 034603 \(2005\)](#).